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Seminário OGTC

Optimization, Graph Theory and Combinatorics

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Graphs whose stability number is easily determined

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Assuming that **P** $\neq$ **NP**, the **NP**-problems (the nondeterministic polynomial problems whose solutions can be guessed and verified in polynomial time) are usually divided into the following complexity classes, the **P**-problems (the problems that can be solved by a polynomial-time algorithm, usually known efficient algorithm) and the **NP**-complete problems (the complexity class which contains many problems that people would like to solve efficiently, but for which no efficient algorithm is known). As it is well known, the determination of the stability number of a graph is **NP**-complete. In this talk, considering the particular case of the determination of the stability number of a graph and a different point of view, we divide the graphs into two classes, the graphs whose stability number is easily determined and the graphs for which we don't know whether or not the stability number is easily determined. It should be noted that if the stability number of any graph is easily determined then **P**=**NP**. The very beginning of this study is the Motzkin-Straus theorem obtained in 1965.

Theorem 1 (Motzskin-Straus, 1965) *Consider a graph G and the quadratic program*

$$f(G) = \frac{1}{2}\max\{x^T A(G)x : x \in \Delta\},$$

where Δ is the simplex $\Delta = \{x \geq 0 : \hat{e}^T x = 1\}$ and $\hat{e}$ is the all one vector. Therefore, if G has a clique number $\omega(G)$, then $f(G) = \frac{1}{2}(1 - \frac{1}{\omega(G)})$.

Later, in the 1990s, Carlos Luz introduced the following convex quadratic program,

$$\upsilon(G) = \max_{y \geq 0} 2\hat{e}^T y - y^T \left(\frac{A_G}{\lambda_{min}(G)} + I\right) y, \tag{1}$$

where $\lambda_{min}(G)$ denotes the least eigenvalue of $A(G)$ and proved that the optimal value of (1) is an upper bound for the stability number of G, that is $\upsilon(G) \leq \alpha(G)$, as well as the necessary and sufficient condition for the equality to be attained. After some algebraic manipulation, this optimal value was related with the (Motzkin-Straus, 1965) result (see Theorem (1)). The talk will continue with several additional results. Namely, for regular graphs, it was proven that the equality holds if and only if the quadratic program (1) admits an optimal solution which is a $(0, \tau)$-regular set, with $\tau = -\lambda_{min}(G)$. We finish with the introduction of the notion of adverse graph and its relation with the recognition of graphs whose stability number are easily determined (also called graphs with convex quadratic stability number or simply Q-graphs) and with a very challenging problem which is still open called the Q-graphs conjecture.

Keywords: stability number; graphs with convex quadratic stability number. MSC 05C50, 15A18.

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