CIDMA CENTRO DE I&D EM MATEMÁTICA E APLICAÇÕES
CENTER FOR R&D IN MATHEMATICS AND APPLICATIONS

universidade de aveiro
Departamento de Matemática

# Seminário OGTC

Optimization, Graph Theory and Combinatorics

7 de maio de 2025 (15h – 16h)

(Sala 11.2.25)

# Graphs of reduced words of a permutation

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Given a Coxeter system $(W, S)$, let $S^*$ be the free monoid generated by the set of words in the alphabet $S$ with concatenation as product. A word $s_{i_1} s_{i_2} \dots s_{i_k}$ in $S^*$ is an expression for $w$ if it represents the element $w \in W$. If $k$ is the minimum among all possible expressions for $w$, we say that $k$ is the *length* of $w$ and the corresponding word is said to be *reduced*. The string of subscripts $i_1 i_2 \dots i_k$ is called a *reduced word* for $w$ and the collection of all reduced words for $w$ is denoted by $R(w)$.

There are three notable graphs associated with the reduced words of a permutation:

- $G(w)$: the vertex set is $R(w)$ where two vertices are connected by an edge if the corresponding reduced words differ by a single Coxeter relation;
- ***Commutations graph*** - $C(w)$: the graph whose vertices are the commutation classes (obtained by contracting commutation edges in $G(w)$), with edges corresponding to long braid relations between classes;
- ***Braid class graph*** - $B(w)$: the graph whose vertices are braid classes (obtained by contracting edges corresponding to long braid relations in $G(w)$), with edges representing short braid relations between the classes.

By introducing the notion of a segment in a word, commutation classes consisting of a single reduced word have been fully characterized. This result has motivated our current investigation into the structure of commutation classes with exactly two reduced words. Our aim is to identify the conditions under which such classes arise and to develop a complete classification for them.

Este seminário é financiado pelo Centro de Investigação e Desenvolvimento em Matemática e Aplicações (CIDMA) através da Fundação para a Ciência e a Tecnologia (FCT), referências UIDB/04106/2020 (https://doi.org/10.54499/UIDB/04106/2020), UIDP/04106/2020 e 2022.03091.PTDC (https://doi.org/10.54499/UIDP/04106/2020).

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