

Seminário OGTC

Optimization, Graph Theory and Combinatorics

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(Sala Sousa Pinto)

Structural and Spectral Properties of Strictly Interval Graphs

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Strictly interval graphs are chordal graphs that are simultaneously strictly chordal and interval. They were introduced by Markenzon and Waga in 2016 and proved to be {2-net, double claw}-free strictly chordal graphs. Strictly chordal graphs, also known as block duplicate graphs, were characterized as the {gem,dart}-free chordal graphs by Golumbic and Peled in 2002. In 2010, new characterizations were presented by Brandstädt and Wagner using the concept of critical clique graphs, which was introduced by Lin et al. in 2000.

In this work, we present a new characterization of strictly interval graphs also given by their critical clique graph structure. This new characterization leads to a simpler linear-time recognition algorithm. Moreover, we define a subclass of strictly interval graphs, called “SI-core graphs”, that are non-split and non-cograph; a significant number of SI-core graphs are Laplacian integral graphs. This work also complements some results presented in 2010 by Kirkland et al..

We present, for any parameters $s, p \geq 2$, the set of SI-core graphs $\mathcal{G}(s, p)$. Firstly, we identify the graphs in $\mathcal{G}(s, p)$ with the minimum number of edges, denoted $G_{\min}(s, p)$, and prove for which values of s and p they are Laplacian integral. Next, we give conditions to insert edges in $G_{\min}(s, p)$ in such a way that the new graphs belong to the same class and are also Laplacian integral. Moreover, for any $s, p \geq 2$, the number of non-isomorphic SI-core graphs in $\mathcal{G}(s, p)$ is given.

Keywords: chordal graphs, Laplacian eigenvalues, graphs and matrices, structural characterizations of types of graphs.

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